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Experiments on laminar flow in curved channels of square section

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Recently Cheng & Akiyama (1970) published a numerical analysis of laminar flow in curved channels of square and rectangular section. Experimental results are presented here for flow in curved channels of square section. The channels were toroidal in shape, and the flow was driven electromagnetically. Various ratios of the channel dimension d to the channel radius of curvature, R, were used to investigate the dependence of friction factor, f, on the Dean number K, and the Reynolds number, Re. For $5\times 10^2 < K < 7\times 10^4$ the formula $(fRe) = 1{\cdot}51\,K^{\frac{1}{2}}$ was found to fit all the results, although R/d was varied from 17·5 down to the low value of 1·75. At lower values of K the analysis of Cheng & Akiyama was approximately validated.

1. Introduction

Flow in curved channels of circular section has been examined theoretically and experimentally by several authors (e.g. Ito 1969, Schmidt 1967), and it is well known that the pressure gradient across the channel due to the channel curvature sets up a secondary flow pattern. Fluid is driven outwards across the core of the flow and returns back along the walls; the secondary flow can be thought of as two vortices of opposite sign each occupying one half of the channel cross-section. The magnitude and pattern of the secondary flow is determined by the Dean number, K, $= (d/R)^{\frac{1}{2}}Re$, where d is the channel diameter, $Re = \rho v_m d/\eta$, ρ is the fluid density, v_m is the mean velocity along the channel, and η is the fluid viscosity. When $K < 1$ the secondary flow is low, the velocity profile of the primary flow is unaffected and the friction factor is the same as for a straight channel. As K increases the maximum in the primary velocity profile is swept towards the outside wall and the friction factor becomes greater than that for a straight channel at the same Reynolds number. The velocity gradients near the wall increase and finally, when $K^{\frac{1}{2}} \gg 1$, the flow exhibits a boundary layer on the wall with thickness of order $d/K^{\frac{1}{2}}$. The returning secondary flow is confined within the boundary layer, the secondary flow in the core is directed purely outwards, and the primary velocity in the core becomes a function of distance measured outwards across the core.

Relatively little has been published on flow in curved channels of rectangular section, but in a recent paper Cheng & Akiyama (1970) calculated numerically laminar forced convection heat transfer in rectangular channels. They considered channels with various ratios of width to height at values of K up to 500. They calculated values of both the friction factor $f[= (dp/dz)/(2\rho v_m^2/d)]$, and the Nusselt number Nu. It is convenient to express the friction factor as F, $= fRe$, and among other results they presented a graph of F_c/F_s versus K, where the subscripts c and s refer to curved and straight channels. On the same graph, which is shown here as figure 1, they included the results of a boundary-layer analysis by Mori & Uchida (1967), and some experimental results of Ludweig (1951), who made measurements on a helically coiled square channel rotating around its axis.

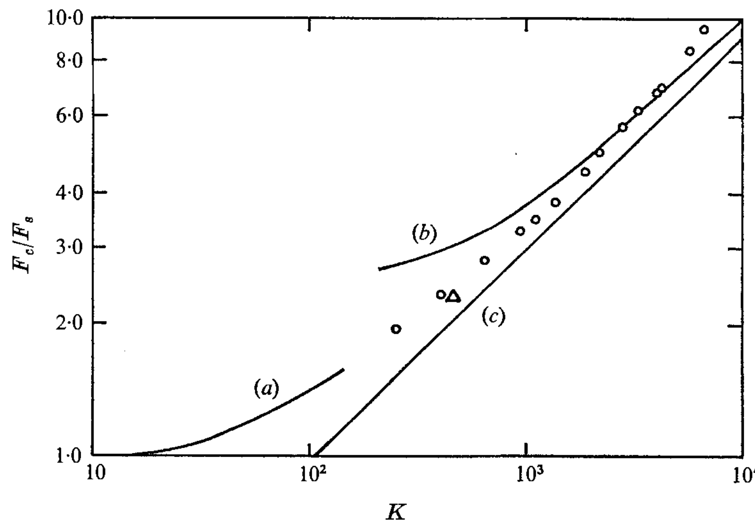

FIGURE 1. Comparison of results for friction factor from published theoretical and experimental work. (a) Calculated curve of Cheng & Akiyama (1970). (b) and (c) Results of boundary-layer analysis by Mori & Uchida (1967). $\triangle$, single point at high Dean number calculated by Cheng & Akiyama; $\bigcirc$, experiments of Ludweig (1951). (The figure is taken from Cheng & Akiyama (1970).)

2. Experimental method

If the flow in a curved channel is pressure driven and the channel length is greater than $2\pi R$, the channel must take the form of a helix. In order to ensure fully developed secondary flow, a helix of several turns is usually used for experiments. This may limit the ratio of R/d which can be tested, and little data for $R/d < 10$ has been published. Fortunately perhaps, most theoretical analyses of flow in curved channels assume $R/d \gg 1$. One method of escaping from the practical difficulty is to drive the flow electromagnetically. Body forces can replace pressure gradients, and it is possible for the channel to take the form of a torus, as shown in figure 2. In particular if the channel is square, the curved walls can be made conducting and the plane walls insulating. The fluid may then be driven by the interaction of a uniform applied magnetic field B along the torus axis with a current I passed between the conducting walls. This geometry was used in an apparatus in which the conducting walls were copper, the insulating walls perspex, and the fluid mercury.

The apparatus was used to investigate boundary-layer behaviour at high values of the Dean number K, or the Hartmann number $M = dB\sigma^{\frac{1}{2}}/2\eta^{\frac{1}{2}}$, where σ is the electrical conductivity of the fluid. It is shown by Baylis & Hunt (1971) that when $(K/M^2)^2\,(d/R) \ll 1$ the flow is similar to that in a straight channel, the secondary flow is suppressed, and there are boundary layers of $O[d/2M]$ on the plane walls. The current I is confined to these layers, which are created by the electromagnetic driving mechanism of the flow. For the flow to resemble a simple pressure-driven flow, the current must be uniform across the channel height, and the current must in effect be squeezed out of the side wall boundary layers. This will occur when the boundary-layer thickness is $\ll d/2M$, and hence when $K^{\frac{1}{2}}/M \gg 1$. A simple order of magnitude analysis of the boundary-layer equations, similar to that given in Baylis & Hunt (1971), shows that the ratio of v_r to v_θ is now $O[(d/r)^{\frac{1}{2}}]$, the layer thickness is $O[d/K^{\frac{1}{2}}]$, and the condition for current uniformity is in fact $(K/M^2) \gg 1$. $\partial v_\theta/\partial r$ is taken here to be $O[v_\theta/d]$ rather than $O[v_\theta/r]$.

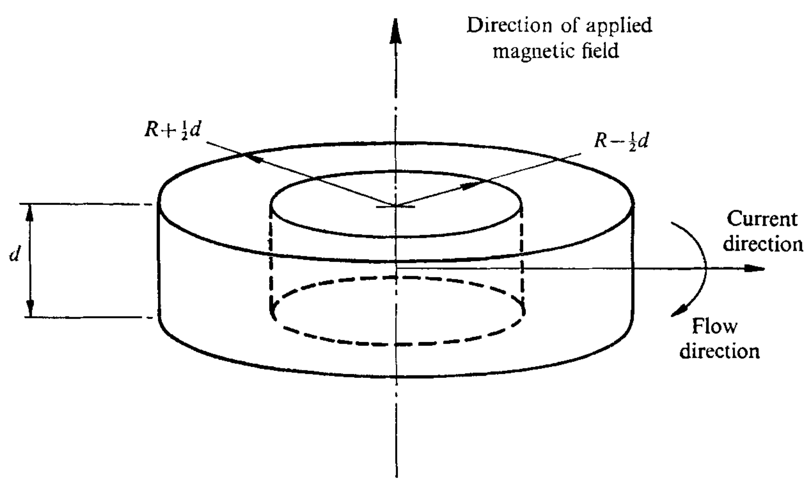

FIGURE 2. The geometry of the flow.

In order to calculate F and K it is necessary to know v_m, the mean velocity. Double integration of Ohm's law over the channel cross-section gives

$$dBv_m = \Delta\phi - \frac{I}{2\pi\sigma d}\ln\left(\frac{2R+d}{2R-d}\right)$$

and hence v_m can be calculated from a measured value of $\Delta\phi$, the potential difference between the conducting walls. The total force driving the fluid is simply IBd, and hence it follows that in terms of non-dimensional parameters $F \equiv JM^2$, where J is a non-dimensionalized current $I/(\pi dRB\sigma v_m)$. For a straight channel with laminar flow $F = 14{\cdot}2$ when the channel has square section (Cornish 1928).

3. Results

Experiments were carried out with $R/d = 17{\cdot}5$, $8{\cdot}5$, 4 and $1{\cdot}75$; in all cases $(R+\frac{1}{2}d)$ was 70 mm. Values of B between $0{\cdot}05$ and $0{\cdot}4$ Tesla and currents between $0{\cdot}1$ and 110 amps were used, while $\Delta\phi$ lay between 1 and $10^4\,\mu$V. Each experiment was carried out at a fixed value of B, and hence of M, and $\Delta\phi$ was measured for a range of values of I. A typical set of results is shown in figure 3 as a plot of P^2 versus K/M^2, where $P = \frac{1}{2}JM$. For $(K/M^2)^2 \ll 1$, P^2 is equal to a constant which tends to unity as M becomes large. For $(K/M^2) > 1$ the data points tie along a line of slope unity, which implies that $F \propto K^{\frac{1}{2}}$. Eventually the points leave the diagonal line and follow a new line of greater slope: the flow is then presumably turbulent. Reynolds numbers at the point of intersection of these two lines were calculated, and were correlated approximately over the range of the experiments by

$$Re_c = 5170\{M(d/R)^{\frac{1}{2}}\}^{0{\cdot}58}.$$

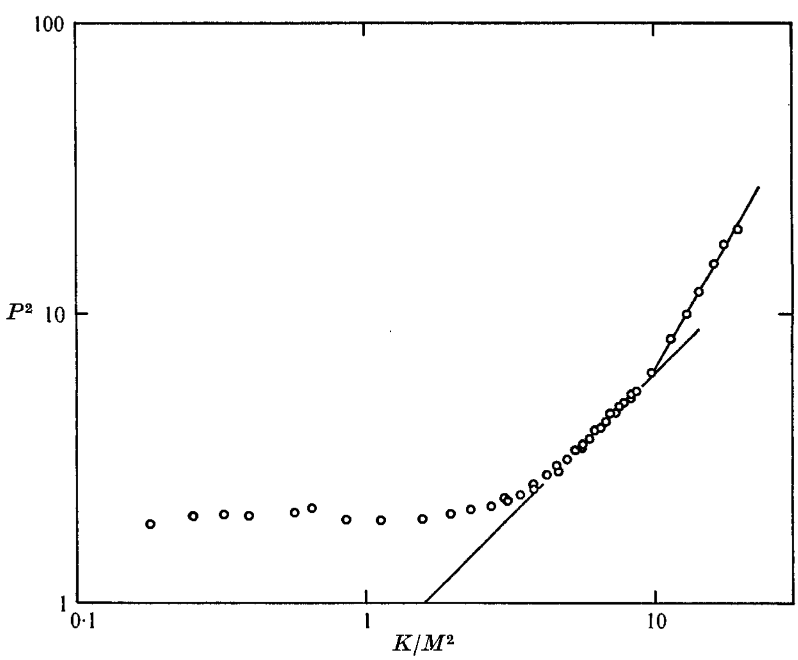

FIGURE 3. Plot of P^2 versus K/M^2 for experiment with $M = 16{\cdot}3$, $R/d = 17{\cdot}5$ showing the Hartmann flow, high secondary flow, and turbulent flow régimes.

The highest experimental value of Re_c was $1{\cdot}04 \times 10^5$, and it is clear that both channel curvature and applied magnetic field have a stabilizing effect on the flow. At the transition to turbulence there was no evidence of unsteadiness, and the transition was smooth: it may not abruptly affect the whole flow, and some approximate measurements of the velocity profile by voltage probes in the upper insulating wall (Baylis 1966) indicated that the profile was most affected near the inner wall. As M increases the transition occurs at lower values of P, and hence the length of the diagonal portion of plots such as figure 3 decreases. The diagonal portion will be called the high secondary flow régime, and is the one relevant to figure 1.

In figure 4 the experimental results for the high secondary flow régime are displayed as plots of F_c/F_s versus K for the four different values of R/d. Points corresponding to turbulent flow were omitted, the criteria for turbulence being a value of P greater than that at the intersection of the two lines on graphs such as figure 3. Points corresponding to 'Hartmann' flow in which P is constant, or the transition region between Hartmann flow and high secondary flow were also omitted, the criteria for such points being $K/M^2 < 4$. Finally, the few points for which $\Delta\phi$ was $< 10\,\mu V$ were omitted because $\Delta\phi$ could only be measured to $\sim 1\,\mu V$. In figure 4 the ordinate scale refers to the points with $R/d = 17{\cdot}5$, and the empirical line $F_c/F_s = 0{\cdot}107\,K^{\frac{1}{2}}$ has been drawn in. For the other values of R/d the ordinate has been shifted, but the same line has been drawn in for each value of R/d so that comparisons can be made.

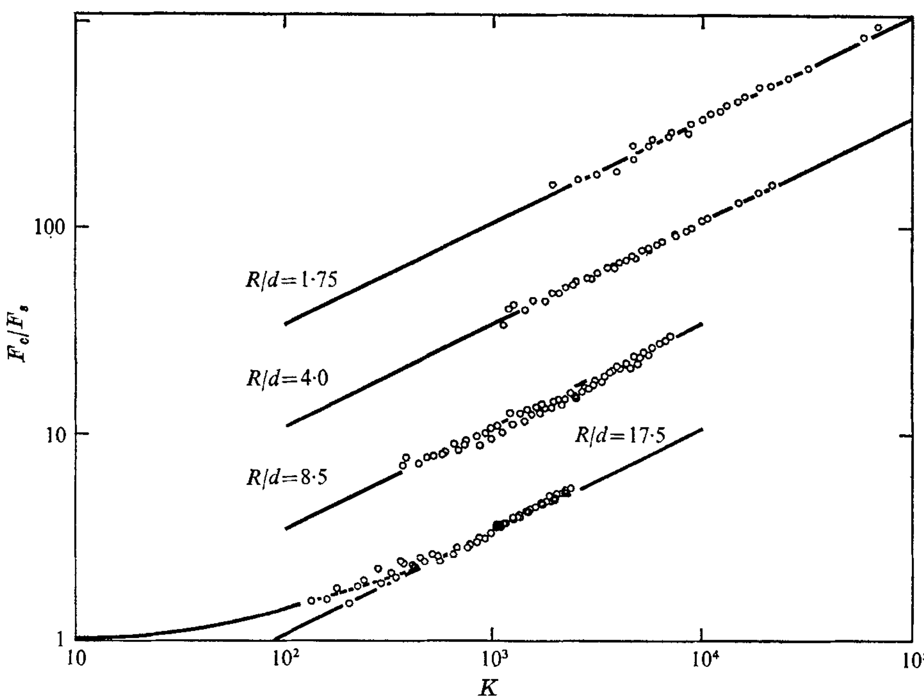

FIGURE 4. Experimental results plotted as F_c/F_s versus K. The line $F_c/F_s = 0{\cdot}107\,K^{\frac{1}{2}}$ is drawn in for each value of R/d. The curve at lower left is taken from the calculated data of Cheng & Akiyama (1970) shown in figure 1.

4. Discussion

Several interesting conclusions can be drawn from figure 4. First, the dependence of F_c on $K^{\frac{1}{2}}$ is clearly demonstrated, and is shown to persist for values of K at least up to 7×10^4. The constant $0{\cdot}107$, which gives the best fit to the points, fits the results of Ludwieg very well. The value of the constant is also very close to the value 0·10 calculated by Ito (1969) for flow in curved channels of circular section. The constant does not seem to depend on R/d, even though the minimum value of R/d was as low as 1·75. This is perhaps unexpected, and analytical confirmation would be interesting. The values of M for $R/d = 17{\cdot}5$ were $\sim 2, 4, 8$ and 16, and for each succeeding value of R/d the four values of M were doubled. On each plot the transition to Hartmann flow occurred at various positions along the plot depending on the value of M. The criteria $K/M^2 < 4$ effectively removed the points which would otherwise have lain above the plots, and hence can be regarded as a successful empirical criteria for the appearance of electromagnetic effects in F_c. On this basis the points at the lower end of the $R/d = 17{\cdot}5$ plot are probably genuine, and represent departure from the $K^{\frac{1}{2}}$ dependence of F_c. For comparison with these points the theoretical curve of Cheng & Akiyama (1970) is drawn in from figure 1. The scatter of the points is rather large, but reasonable agreement with theory is shown.

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